

# Moduli stabilisation and de Sitter string vacua from magnetised D7 branes

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**ABSTRACT:** Anomalous  $U(1)$ 's are ubiquitous in 4D chiral string models. Their presence crucially affects the process of moduli stabilisation and cannot be neglected in realistic set-ups. Their net effect in the 4D effective action is to induce a matter field dependence in the non-perturbative superpotential and a Fayet-Iliopoulos D-term. We study flux compactifications of IIB string theory in the presence of magnetised D7 branes. These give rise to anomalous  $U(1)$ 's that modify the standard moduli stabilisation procedure. We consider simple orientifold models to determine the matter field spectrum and the form of the effective field theory. We apply our results to one-modulus KKLT and multi-moduli large volume scenarios, in particular to the Calabi-Yau  $\mathbb{P}^4_{[1,1,1,6,9]}$ . After stabilising the matter fields, the effective action for the Kähler moduli can acquire an extra positive term that can be used for de Sitter lifting with non-vanishing F- and D-terms. This provides an explicit realization of the D-term lifting proposal of [1].

**KEYWORDS:** Flux compactifications, Superstring Vacua, D-branes.

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## 1. Introduction

Significant progress has been made recently regarding the supersymmetry breaking and moduli stabilisation problems of string compactifications (for recent reviews see [2, 3]). In particular, IIB string flux compactifications have provided concrete models of moduli stabilisation in which the scale of supersymmetry breaking can be calculated as well as the relevant soft breaking terms in the effective action.

This has been achieved independently of the details and location of the chiral fields of the Standard Model. This is in part because of the ‘modular’ structure inherent in type II string models. There are global bulk issues such as moduli stabilisation, inflation and supersymmetry breaking, and there are local brane issues regarding the gauge group, spectrum of chiral fields, etc. Usually the two types of issues can be approached independently of each other. Separating global questions from local ones in a systematic construction of realistic models was proposed in [4] and called the ‘bottom-up’ approach to string model building. For relevant progress on the model building side see [5, 6].

This procedure is very efficient in the sense that once a global problem, such as moduli stabilisation, has been solved, then a realistic D-brane construction in terms of D-branes at singularities or magnetised D7 branes can be attached to the compactification manifold to make it into a realistic model. However at the end of the day we have to consider the two parts together in order to control the low-energy nature of soft supersymmetry breaking terms, reheating, local and global symmetries, etc. We therefore should investigate the effects that chiral D-brane models have in the effective action for the moduli fields and if it is needed to incorporate them in the moduli stabilisation procedure.

One of the generic properties of chiral D-brane models is the presence of anomalous U(1)’s. In a typical construction, the chiral matter of the spectrum naturally induces anomalies for some of the U(1) gauge fields. The anomaly is cancelled by the standard Green-Schwarz mechanism with the net effects of giving a mass to the corresponding gauge field, a ‘charge’ to the modulus field corresponding to the gauge coupling of the effective field theory, and inducing a Fayet-Iliopoulos D-term proportional to the total charge of the chiral fields. This shows in particular that the non-perturbative terms in the KKLT scenario [7] involving only the Kähler modulus field are not gauge invariant, and therefore that the chiral matter fields must also enter the superpotential in such a way to render it gauge invariant. Thus the effects of anomalous U(1) fields must be taken into account in the moduli stabilisation procedure if we have chiral fields living in D-branes, as is required for instance by the inclusion of the Standard Model.

The fact that anomalous U(1)’s also induce Fayet-Iliopoulos (FI) D-terms can modify the moduli stabilisation procedure. Since D-terms are positive, it was proposed in [1] that they can be used to lift the original KKLT AdS minimum to a de Sitter one in a way consistent with a supersymmetric effective action. See [8–11] for recent discussions of this proposal. More generally there is an important question that emerges here: could the effects induced by anomalous U(1)’s change the successful results regarding moduli stabilisation so far?

In this article we address the issue of moduli stabilisation in the presence of anomalous U(1)’s<sup>1</sup>. Following the standard procedure (and the proposal of [1]) we consider D7 branes with non-vanishing magnetic fluxes. Magnetic fluxes are the standard source of chirality and of the anomalous U(1)’s. In the next section we consider simple orientifold models and determine the corresponding U(1) charges of the different matter fields and compute the FI term. In section 3 we consider simple examples of one modulus KKLT type or

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<sup>1</sup>For a perturbative moduli stabilisation proposal including FI terms see [13].

several moduli large volume type and see how moduli stabilisation is affected by these new ingredients, including a non-perturbative superpotential of the Affleck-Dine-Seiberg type invariant under the anomalous  $U(1)$ . We find that these effects either leave the good features of the model, such as the existence of exponentially large volumes, or modify them to allow de Sitter lifting, depending on the model, the distribution of the D7 branes and magnetic fluxes. We include several appendices with details of some of the calculations, including anomaly cancellation and the FI term.

## 2. D-terms and de Sitter vacua

### 2.1 General considerations

One of the key steps in the KKLT procedure is the lifting of vacuum energy to a positive value. In the original KKLT paper [7], the lifting term arose from an anti-D3 brane localised in a highly warped region of the Calabi-Yau. This type of brane breaks supersymmetry explicitly, and the low energy effective theory cannot be described by the standard  $N = 1$  four dimensional supergravity.

An alternative and more controlled lifting mechanism was proposed in [1]. The idea is to use a D-term generated by magnetic fluxes on D7 branes to provide an additional positive contribution to the scalar potential. This can, under favourable conditions, result in a de Sitter vacuum. The formalism is that of supergravity and the supersymmetry breaking is spontaneous rather than explicit.

One of the reasons the original proposal of [1] ought to be studied in more detail is the observation, pointed out in [8], that in a general  $N = 1$  supergravity theory, there exists a relation between F-terms and D-terms,

$$D = \frac{i}{W} \sum_i (\delta\phi_i) D_i W. \tag{2.1}$$

Here  $W$  is the superpotential,  $f$  the gauge kinetic function,  $D_i W = \partial_i W + W \partial_i K$  the Kähler covariant derivative with  $K$  the Kähler potential and  $\delta\phi_i$  the transformation of the field  $\phi_i$  under the  $U(1)$  generating the D-term. The relation (2.1) follows simply from gauge invariance and holds at any point in field space (except where  $W = 0$ ). It is clear that  $D_i W = 0$  implies  $D = 0$ , making it essentially impossible to uplift an original SUSY vacuum, i.e. to have pure D-term supersymmetry breaking, contrary to the standard global supersymmetry case. The proposal in [1] actually considered both F and D terms to be non-vanishing once matter fields were introduced, but an explicit analysis was not done. This has been done in recent publications [9–11] in the context of the KKLT scenario.

Since the original KKLT scenario is such that the AdS minimum is supersymmetric, it is difficult to simply lift it by a standard D-term. Usually  $D = 0$  can be used to fix the charged matter fields  $\phi_i$  giving the KKLT F-term potential as a function of the moduli fields only, and hence essentially recovering the KKLT result.

On the other hand there exist a large class of models in which moduli stabilisation is achieved in such a way that the AdS minimum is non-supersymmetric [15–17]. In most of these cases the volume is exponentially large with very interesting phenomenological

implications. Since the original F-term is non-zero it is natural to expect that a D-term can be non-zero, contributing a positive term to the scalar potential and therefore leading to the possibility of de Sitter lifting. Therefore the natural context in which to look for vacua that can be uplifted with D-terms is that of nonsupersymmetric large volume compactifications. Another interesting question for these models is whether the addition of D-terms still preserves the fact that the stabilised volume is still exponentially large.

D-terms in string theory [18] have a very interesting structure, since they are generically related to massive and anomalous U(1)s and the Green-Schwarz mechanism, and this relation greatly constraints the form of the D-term potential. We will briefly review this topic in the following subsection, before considering concrete setups in which we will carry out the lifting procedure.

## 2.2 On FI terms, anomalous U(1)s and massive gauge bosons

As explained in [19, 20, 22], turning on internal magnetic flux in the worldvolume of a D7 brane generates an FI term in the four dimensional world-volume theory. This can be seen from several points of view. Firstly, the presence of the magnetic field generates chiral fermions and scalars in the low energy theory. For certain values of the moduli the scalars become massless and the theory supersymmetric. This pattern of supersymmetry breaking/restoration is clearly reminiscent of an FI mechanism. But perhaps the clearest way to see that an FI term is generated is considering the fact that four dimensional couplings of the form  $\int D_2 \wedge F$ , where the 2-form  $D_2$  comes from the reduction of the 4-form RR field  $C_4$ , and  $F$  is the four dimensional gauge field strength, are generated when internal magnetic flux is turned on. This kind of four dimensional coupling generates masses for the corresponding U(1) gauge bosons. The 2-form field  $D_2$  is the four dimensional Hodge dual of an axion zero form  $\phi$ , which is the imaginary part of a chiral superfield modulus  $T$  that parametrises the volume of some 4-cycle. The field  $\phi$  transforms under a U(1) gauge transformation as  $\phi \rightarrow \phi + Q\theta(x)$ , where  $Q$  (the charge of this transformation) is related to the internal magnetic flux on the D-brane. It follows that in order to get the  $\int D_2 \wedge F$  coupling, the Kähler potential of the four dimensional theory must depend on the combination  $T + T^* + QV$ , with  $V$  the vector multiplet corresponding to  $F$  [18]. But in a supersymmetric theory the presence of a gauge boson mass is always linked to an FI term, since both come from the same term in the Lagrangian when expressed in the superfield formulation. For global supersymmetry:

$$\int d^4\theta K(T + T^* + QV) = \left(\frac{\partial K}{\partial V}\right)_{V=0} V|_{\theta^4} + \frac{1}{2} \left(\frac{\partial^2 K}{\partial V^2}\right)_{V=0} (\partial_\mu \phi + A_\mu)^2 + \dots$$

Let us be more precise. Consider the Chern-Simons part of the D-brane action

$$S_{CS} = -\mu_7 \int_{D7} \sum_p i^* C_p \wedge e^{i^* B + \mathcal{F}}. \tag{2.2}$$

Here  $\mathcal{F}$  is a mass dimension two form, and  $\mathcal{F} \equiv 2\pi\alpha' F$  so that  $F$  has mass dimension 0.  $i^*$  denotes the pullback operation. Expanding this action, the coupling to the RR 4-form

$C_4$  is<sup>2</sup>

$$\int_{D7} C_4 \wedge F \wedge F. \tag{2.3}$$

Taking one of the  $F$ 's to be the compact flux  $f$  and the other to be with non-compact indices (denoted by  $F$ ), and reducing  $C_4 = D_2^\alpha \wedge \omega_\alpha + \dots$ , where  $\omega_\alpha$  form a basis for the 2-cohomology of the Calabi-Yau, we have

$$\int_{\Sigma} \omega_\alpha \wedge f \int_{M^4} D_2^\alpha \wedge F. \tag{2.4}$$

Here  $\Sigma$  denotes the 4-cycle wrapped by the D-brane. What is interesting to note here is that since  $D_2^\alpha$  is correlated with the 2-cycle whose volume form is proportional to  $\omega_\alpha$ , its four dimensional Hodge dual, the axion zero form  $\sigma_\alpha = \int_{\Sigma_\alpha} C_4$ , must be related to the four cycle that is Poincaré dual of  $\omega_\alpha$ , namely  $\tilde{\omega}^\alpha$ . Since one defines the Kähler moduli fields as [22]

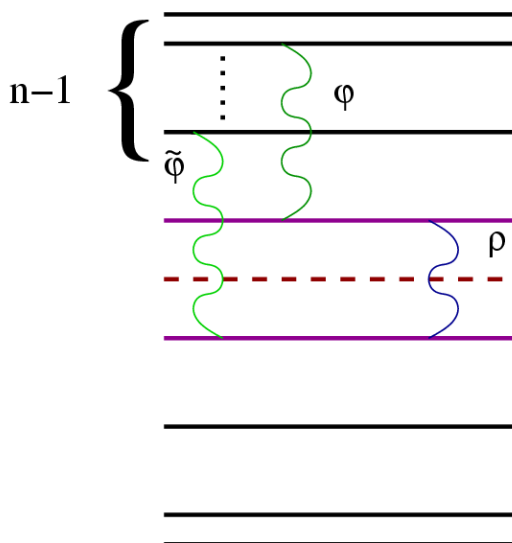
$$T_\alpha = \frac{1}{2\pi(2\pi\sqrt{\alpha'})^4 g_s} \left( \int_{\Sigma_\alpha} \sqrt{g} d^4x + i\sigma_\alpha \right), \tag{2.5}$$

we see that the Kähler moduli fields that get charged under the U(1) are those parameterising volumes of a four-cycles that have non-zero intersection with the two-cycle where the magnetic field is supported. Also, since the coupling (2.4) gives a mass for the U(1) gauge boson supported on the brane with internal magnetic flux, it follows from supersymmetry that whenever magnetic flux is turned on in a D7-brane, both a Fayet-Iliopoulos term<sup>3</sup> and a mass for the corresponding U(1) gauge boson are generated. However, it is important to emphasise that this magnetic flux does not generically generate a charge for the  $T$  modulus whose vev parametrises the volume of the 4-cycle wrapped by the D7 with internal magnetic flux. This field  $T$  only gets charged if the corresponding 4-cycle has self-intersections [20, 22].

The fact that a given field  $T_\alpha$  gets charged does not mean that the  $U(1)_a$  associated to the brane supporting magnetic flux is anomalous. Such a U(1) will be anomalous if there exists a term in the four dimensional low energy effective action of the form  $\sigma_\alpha \text{Tr } F_b \wedge F_b$ , where  $F_b$  is the field strength of some  $U(1)_b$  or  $SU(N_b)$  gauge group present in the construction, or  $\sigma_\alpha \text{Tr } R \wedge R$ ,  $R$  being the Ricci form. The former couplings give rise to mixed  $U(1)_a - SU(N_b)^2$  and mixed  $U(1)_a - U(1)_b^2$  anomalies in the low energy theory (note that the cubic  $a = b$  case is a particular case of this). The latter couplings give rise to gravitational anomalies in the low energy theory. There will be mixed gauge anomalies for a given U(1) whenever any D7 brane wraps a four-cycle that is charged under this U(1). The cubic anomalies will arise as a particular case of this, either because the cycle has self-intersections or has a zero intersection with its orientifold image. The gravitational anomaly will typically arise when the corresponding D7 brane has non-zero intersection with the orientifold. Note that every anomalous U(1) will automatically be massive, but a massive U(1) need not be anomalous.

<sup>2</sup>We will not consider couplings to the NSNS field  $B_2$  in what follows.

<sup>3</sup>Note that since this FI term is a field dependent quantity, it can indeed be zero in some regions of the moduli space. If this was not the case, it would be impossible to have magnetised brane constructions preserving supersymmetry.



**Figure 1:** This is a representation of the basic set-up. A set of  $n$  D branes is such that one of them is magnetised leading to a  $U(n - 1) \times U(1)$  gauge group. The dotted lines represent the orientifold plane. Chiral fields are  $\varphi$  in the fundamental of  $SU(N_c)$  (with  $N_c = n - 1$ ) corresponding to strings going from the non-abelian set of branes to the magnetised brane,  $\tilde{\varphi}$  in the anti-fundamental, representing strings with endpoints in the non-abelian set of branes and the orientifold image of the magnetised brane and  $\rho$  corresponding to strings with endpoints at the magnetised brane and its orientifold image.

Consider a setup in which magnetic flux has been turned on in the world-volume of a D7 brane wrapping a 4-cycle with Kähler modulus  $T_F$ . The D-term potential arising from this setup is

$$V_D = \frac{g^2}{2} \left( \frac{Q_\alpha}{4\pi^2} \partial_{T_\alpha} K + \sum_i q_i \phi_i K_{\phi_i} \right)^2. \quad (2.6)$$

where the  $T_\alpha$  are all the Kähler moduli charged under the anomalous  $U(1)$ , the  $\phi_i$  are the unnormalised open string fields charged under the same  $U(1)$ , and  $K_\phi = \partial K / \partial \phi$ . One can get the expression for the gauge coupling constant  $g^2$  from the dimensional reduction of the DBI action. The result [22] is

$$g^{-2} = \text{Re}T_F - f_F \text{Re}S. \quad (2.7)$$

Here  $f_F$  is a certain magnetic flux dependent factor. In the large  $T_F$  limit which we will be concerned with throughout the paper, we may neglect the  $\text{Re}S$  contribution to  $g^{-2}$ .

### 2.3 General setup and spectrum

One of the aims of this paper is to study under which circumstances it is possible to get D-term lifting in string models. As already emphasised, it is better to start from a non-supersymmetric vacuum in order to get the lifting. Given this, a natural context to work in

are the large volume models [16, 17], in which vacua have naturally non vanishing F-terms. One could also attempt to apply D-term lifting on KKLT vacua, since there may still be minima in which both the D-term and the F-terms are non-vanishing. We can therefore imagine a few different scenarios. In the next sections we will consider the following:

- (i) One may consider the simplest model with one Kähler modulus  $T$ , with a nonperturbative superpotential fixing the modulus, and turn on magnetic fluxes such that  $T$  becomes charged.
- (ii) Alternatively, one can consider a model with at least two Kähler moduli for which large volume minima have been found [16]. Then one may turn on magnetic fluxes on the exponentially large 4-cycle which determines the volume so that the corresponding modulus becomes charged, without the need of including a nonperturbative superpotential for it.
- (iii) One can also consider a model with more than one modulus and turn on magnetic fluxes on branes wrapping one of the ‘small’ 4-cycles which is nonperturbatively stabilised. There are two sub-cases here, the first being that the cycle becoming charged under the  $U(1)$  is the large 4-cycle, and the second that the small 4-cycle becomes charged.

There is one further choice that must be made — whether the D7 branes lie on top of orientifold planes or away from them. The former choice ensures that the local dilaton charge is cancelled but introduces new complications due to extra matter fields being present. We consider both possibilities.

In each one of these constructions, the local setup we will consider is as follows. We will take a set of D-branes and O-planes, typically on top of each other in order to cancel local tadpoles.<sup>4</sup> We consider magnetic flux turned on in one of the branes in the stack. The presence of this magnetic flux will generate an FI term in the four dimensional theory, as explained in the previous section. Also, the magnetic flux will be responsible for the appearance of chiral superfields in the overlapping region with the magnetised two-cycle.

The gauge group will be of the form  $G \times U(1)_F$ , where the  $U(1)_F$  factor corresponds to the branes where we have put magnetic flux<sup>5</sup> (we will call it brane or stack  $F$ ) and  $G$  can be either<sup>6</sup>  $SO(N)$ ,  $USp(N)$  or  $U(N)$ , and lives in the remaining stack of branes (stack  $G$ ). In this situation, one expects the following set of fields to appear:

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<sup>4</sup>One can also consider the branes being away from the orientifold plane but, as emphasised in [22], this potentially leads to F-theory corrections that are beyond the scope of this work.

<sup>5</sup>For some orientifold projections, the Chan-Paton projection will require to put the same units of magnetic flux in more than a stack of branes in order to get the  $U(1)$  factor.

<sup>6</sup>This will depend on the orientifold projection over the Chan-Paton degrees of freedom. Generically, if the stack of branes has no magnetic flux one will get  $SO(N)$  or  $USp(N)$ , with  $N$  related to the number of branes in the stack in a model-dependent way, and the choice of gauge group depends on the choice of orientifold projection. The  $U(N)$  case can arise either if one considers the branes to be away from the orientifold projection, or if the stack of branes is also magnetised, with an internal magnetic flux that is different from the flux of the D-brane giving rise to the  $U(1)_F$  gauge group.



- A set of fields transforming in the  $(+1_F, \overline{\square}_G)$  under  $U(1)_F \times G$ , that live between the brane  $F$  and the stack  $G$ . We will call these fields  $\varphi_i$ .
- Another set of fields transforming in the  $(+1_F, \square_G)$  under  $U(1)_F \times G$ , that live between the magnetised brane  $F$  and the orientifold image of  $G$ . We will denote these fields by<sup>7</sup>  $\tilde{\varphi}_i$ .
- A set of fields  $\rho$  with charge  $\pm 2$  under  $U(1)_F$ . These fields live between the brane  $F$  and its orientifold image  $F^*$ .

More generically, one can consider a stack of  $F$ -branes, all of them with the same internal magnetic field. In this case, the fields  $\varphi$  transform under the  $(\square_F, \overline{\square}_G)$ , the fields  $\tilde{\varphi}$  (if present) transform under the  $(\square_F, \square_G)$  and the  $\rho_{ij}$  transform under symmetric and antisymmetric representations of  $U(N_F)$  (note  $N_F$  is not necessarily equal to the number of  $F$ -branes). As we will see now, one can constrain the number of these fields from anomaly cancellation arguments.

## 2.4 Anomaly cancellation constraints on the spectrum

The setup we are considering is a set of D7-branes on top of a set of O-planes, and we will turn on magnetic flux on the former. This magnetic flux induces a tadpole for a lower dimensional brane charge, that induces a chiral anomaly in the world volume of the branes. Also, gravitational anomalies will generically be present. We will compute the spectrum on these branes using anomaly cancellation arguments.

Consider two D7 branes  $a$  and  $b$ , wrapping different 4-cycles  $\Sigma_a$  and  $\Sigma_b$ , and overlapping over a 2-cycle where both of them support magnetic flux. This gives rise to a number of chiral fermions, given by the index

$$\begin{aligned}
 I_{ab} &= \int_{\Sigma_a} \text{PD}_Y(\Sigma_b) \wedge F_a - \int_{\Sigma_b} \text{PD}_Y(\Sigma_a) \wedge F_b \\
 &= \int_{\Sigma_b} \text{PD}_Y(\Sigma_a) \wedge F_a - \int_{\Sigma_a} \text{PD}_Y(\Sigma_b) \wedge F_b
 \end{aligned}
 \tag{2.8}$$

where we define  $\text{PD}_A(B)$  to be the Poincaré dual of the cycle  $B$  taken with respect to the (sub)manifold  $A$ . In order to make contact with standard anomaly inflow arguments, we would like to define this index as a product between D7 and D5 charges, such that the product is bilinear in them. To do this we define the total class of a D7 with magnetic flux as [6]

$$[D7_a] = \text{PD}_Y(\Sigma_a) + \text{PD}_Y(\text{PD}_{\Sigma_a}(F_a)).
 \tag{2.9}$$

This definition includes the D7 charge specified by the (co)homology class of the 4-cycle  $\Sigma_a$  wrapped by the D7, and the D5 charges generated by the world-volume magnetic flux.<sup>8</sup>

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<sup>7</sup>Note that there can be situations in which  $\square_G = \overline{\square}_G$  and hence the  $\tilde{\varphi}$  fields can be considered as the antiparticles of the  $\varphi$ , so that we do not need to consider them in the D-term potential.

<sup>8</sup>There is also a D3 charge that we do not mention since is irrelevant in the following.

Then we define the inner product between two D7 classes as

$$\begin{aligned}
 [D7_a] \cdot [D7_b] &\equiv \int_Y [\text{PD}_Y(\Sigma_a) + \text{PD}_Y(\text{PD}_{\Sigma_a}(F_a))] \wedge [\text{PD}_Y(\Sigma_b) - \text{PD}_Y(\text{PD}_{\Sigma_b}(F_b))] \\
 &= \int_{\Sigma_b} \text{PD}_Y(\text{PD}_{\Sigma_a}(F_a)) - \int_{\Sigma_a} \text{PD}_Y(\text{PD}_{\Sigma_b}(F_b)). \tag{2.10}
 \end{aligned}$$

This intuitive definition is bilinear in the brane charges and applies in the simplest cases. In the general case several subtleties arise and one can define such a product in a much more formal (and complicated) way [6, 23, 24]. Since we do not want to overload the paper with mathematical jargon, and moreover since the only property of  $I_{ab}$  that will be relevant in what follows is bi-linearity in the D5 and D7 charges (that is also present in the complete formulae), we will use the simplified expression (2.10) and refer the interested reader looking for a more formal definition of the index to the above-mentioned papers.

As already mentioned, the total number of chiral multiplets charged under a given gauge group will be given by the multiplets  $\varphi$ ,  $\tilde{\varphi}$  appearing between the brane (or stack of branes) supporting the gauge group and other (stacks of) branes appearing in the configuration (and their orientifold images), plus a set of charge  $\pm 2$  fields (that we denote generically by  $\rho$ ) that can appear between the brane (or stack of branes) and its orientifold image. The number of  $\varphi$ ,  $\tilde{\varphi}$  fields in the configurations will be given by the expressions (2.8) or (2.10), while the number of  $\rho$  fields is generically model dependent. However, as we will see now, standard anomaly cancellation arguments already used in [25] can easily give us this number.

To start with, let us consider a configuration with a  $U(N_F) \times U(N_G)$  gauge group. Generically we will have  $N_F = 1$  and  $G \neq U(N_G)$ , but since what we want to compute is the number of  $\rho$  fields, whose number should only depend on the relation between the brane  $F$  and the orientifold and thus should be independent of  $G$ , and moreover most of the orthogonal and symplectic groups do not give rise to anomalies, this calculation will suffice to compute the actual number of  $\rho$  fields regardless of the particular situation.

Given the spectrum, the kind of anomalies one can expect are:  $SU(N_F)^3$ ,  $U(1)_F - SU(N_G)^2$ ,  $U(1)_F - SU(N_F)^2$ ,  $U(1)_F - U(1)_G^2$ ,  $U(1)_F^3$  and gravitational. Out of all these, only the first two must be cancelled without any help from a Green-Schwarz mechanism since there are no  $U(1)$ s involved. In this section we will see how the matter content required by the cancellation by the cubic  $SU(N_F)^3$  anomalies uniquely fixes the number (and charge) of the  $\rho$ -fields. We show in an appendix how this matter content is precisely the one required for the rest of the anomalies to be cancelled, together with a Green-Schwarz mechanism.

To get a  $U(N_F) \times U(N_G)$  gauge group one has to put different magnetic flux in two different stacks of branes. Let us assume that we have a set of  $N = n + m$  branes, so that we put a flux of the form  $F_n$  in the  $n$  branes (that wrap a 4-cycle  $\Sigma_n$ ) and flux of the form  $F_m$  units in the  $m$  branes (that wrap a four cycle  $\Sigma_m$ , not necessarily different from  $\Sigma_n$ ). More concretely, using the notation defined in (2.9)

$$[D7_i] = \text{PD}_Y(\Sigma_i) + \text{PD}_Y(\text{PD}_{\Sigma_i}(F_i)). \tag{2.11}$$

with  $i = n, m$ . Tadpole cancellation implies

$$n([D7_n] + [D7'_n]) + m([D7_m] + [D7'_m]) - [O7] = 0. \quad (2.12)$$

with  $[O7]$  the class of the orientifold plane, defined analogously to (2.9) and where we have included the number of orientifold planes and charge of the orientifold compared to that of a D7 in the definition of  $[O7]$ .  $[D7'_i]$  the orientifold image of  $[D7_i]$ , given by

$$[D7'_i] = \text{PD}_Y(\Sigma_i) - \text{PD}_Y(\text{PD}_{\Sigma_i}(F_i)). \quad (2.13)$$

The gauge group on the branes will be  $U(N_F) \times U(N_G)$ , with  $n/N_F = m/N_G = b \in \mathbf{Z}^+$ . Now, we know that tadpole cancellation conditions must imply the cancellation of cubic  $SU(k)$  anomalies for  $k = n, m$ , without any contribution from GS terms.

Let us consider the spectrum. There is a set of  $\varphi$  fields transforming in  $(\square_{N_F}, \bar{\square}_{N_G})$ , and a set of  $\tilde{\varphi}$  fields transforming in  $(\square_{N_F}, \square_{N_G})$ . The number of them is given by<sup>9</sup>

$$\#\varphi \equiv I_{nm} = [D7_n] \cdot [D7_m] \quad (2.14)$$

$$\#\tilde{\varphi} \equiv I_{nm'} = [D7_n] \cdot [D7'_m]. \quad (2.15)$$

There can also be symmetric and antisymmetric representations<sup>10</sup>. The number of these fields has to be related to the numbers  $I_{kk'}$  and  $I_{kO}$ , with  $k = n, m$ , defined as

$$I_{kk'} \equiv [D7_k] \cdot [D7'_k], \quad (2.16)$$

$$I_{kO} \equiv [D7_k] \cdot [O7]. \quad (2.17)$$

Thus, we express the number of symmetric and antisymmetric representations of  $N_F$  as

$$\#\square_{N_F} = \alpha_s I_{nn'} + \beta_s I_{nO} \quad (2.18)$$

$$\#\bar{\square}_{N_F} = \alpha_a I_{nn'} + \beta_a I_{nO}. \quad (2.19)$$

A similar formula holds for  $N_G$ . Multiplying eq. (2.12) by  $[D7_n]$  on the left we get

$$nI_{nn'} + m(I_{nm} + I_{nm'}) - I_{nO} = 0. \quad (2.20)$$

Since  $m(I_{nm} + I_{nm'})$  is precisely  $b$  times the anomaly produced by the  $\varphi, \tilde{\varphi}$ , it follows that

$$b[\mathcal{A}_s(\#\square) + \mathcal{A}_a(\#\bar{\square})] = nI_{nn'} - I_{nO}. \quad (2.21)$$

---

<sup>9</sup>The absolute value of this index signals the net number of fermions and the sign denotes its four dimensional chirality. Since in field theory one typically choses all fermions to be of the same chirality, one takes the convention in which a positive sign for  $I_{ab}$  indicates the existence of  $|I_{ab}|$  fermions with positive chirality and charges  $(1_a, -1_b)$ , whereas a negative sign for  $I_{ab}$  implies the existence of  $|I_{ab}|$  fermions with positive chirality and charges  $(-1_a, 1_b)$ .

<sup>10</sup>In some constructions it is possible that tensorial representations can also appear in the confining sector. In those cases, the non-perturbative superpotential is not known and they would require a further analysis which is beyond the scope of this paper (for an explicit derivation of the Affleck-Dine-Seiberg superpotential from non-perturbative string effects see [26]).

with  $\mathcal{A}_{s(a)}$  is the anomaly produced by the (anti)symmetrics. Given that  $\mathcal{A}_s = N_F + 4$ ,  $\mathcal{A}_a = N_F - 4$ , substituting (2.18) and (2.19) into (2.21), and requiring the number of fields to be independent of  $N_F$ , we get

$$\#\square_{N_F} = \frac{1}{2} \left( I_{nn'} - \frac{1}{4b} I_{nO} \right), \tag{2.22}$$

$$\#\square_{N_F} = \frac{1}{2} \left( I_{nn'} + \frac{1}{4b} I_{nO} \right). \tag{2.23}$$

These (anti)symmetrics are, from the  $U(1)_{N_F}$  point of view, a set of charge  $\pm 2$  fields that we will call  $\rho$ .

There are  $N_F(N_F + 1)/2 \times \#\square_{N_F}$  of them coming from the symmetric representation and  $N_F(N_F - 1)/2 \times \#\square_{N_F}$  coming from the antisymmetric one. The total number of  $\rho$  fields is then given by

$$\#\rho = \frac{N_F^2}{2} I_{nn'} - \frac{N_F}{8b} I_{nO} \tag{2.24}$$

Note that, depending on the orientifold content, this number can be positive, negative or zero. In the following we will take the convention that the four dimensional chirality of the corresponding fermion is fixed and the sign of the number of  $\rho$ s is equal to the charge of the field. Finally, we insist in the fact that since the number of  $\rho$  fields should be independent of the gauge group  $G$ , we will also have the number of  $\rho$  fields given by (2.24) in the general situation with  $SO(N)$  or  $USp(N)$  gauge groups.

Summarising, we have found that there are in general three types of fields, fundamentals  $\varphi_i, \tilde{\varphi}_j$  and singlet fields under the (generically non-abelian) group  $G$ ,  $\rho_{ij}$ , where  $i, j$  are flavour indices.

Given this, one can always choose to fix the charges both of the  $\varphi$  and  $\tilde{\varphi}$  to be  $+1$ . Depending on the model under consideration, the field  $\rho$  can have charge  $+2$  or  $-2$  (cf (2.24)). Also, anomaly cancellation arguments imply that the charge of  $T$  has the opposite sign to the charge of both  $\varphi$  and  $\tilde{\varphi}$ . Given this, the relative sign between the FI term and the  $\varphi, \tilde{\varphi}$  dependent parts of the D-term is uniquely determined by the sign of  $\partial_T K$ . Each one of these possibilities will lead to different behaviour regarding the D-terms and potential de Sitter lifting. We will consider all these cases in the following section.

### 3. Explicit constructions

We now study how D-term lifting can be explicitly realised <sup>11</sup> in the various cases listed in the previous section.

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<sup>11</sup>That is, in the context of a particular Calabi-Yau compactification with one or more Kähler moduli. We do not give concrete D-brane configurations with magnetic fluxes but assume that a configuration of branes and fluxes with the properties described in the previous section has been achieved and then study its implications.

### 3.1 One modulus case

The simplest case to consider is the one Kähler modulus case, studied in the original KKLT paper [7]. We will discuss it first for illustration purposes.

We will assume that the 4-cycle corresponding to the only Kähler modulus  $T$  carries D7 branes with a gauge theory undergoing gaugino condensation. We will also assume that the stack of branes under consideration does not intersect any other stacks of D3 branes, to avoid the appearance of additional massless matter in the low energy theory (this possibility has been recently discussed in [27]).

We consider one of the D7s and turn on a U(1) magnetic flux on a 2-cycle which belongs to the 2-homology of the 4-cycle the branes wrap. The fields  $\rho$  corresponding to strings stretching between the magnetised D7 and its orientifold image have charge  $\pm 2$  under the U(1) gauge group.<sup>12</sup> There are also quark fields  $\varphi, \tilde{\varphi}$  with charge  $+1$  which correspond to open strings stretched between the magnetised brane and the unmagnetised D7s and their orientifold images.

If an appropriate topological condition is satisfied (that the cycle wrapped by the D7 branes intersects itself over the 2-cycle with magnetic flux), the modulus  $T$  will become charged under the U(1) and a D-term potential is induced of the form

$$V_D = \frac{1}{T + T^*} ((\partial_T K)\delta T + (\partial_\rho K)\delta\rho + (\partial_\varphi K)\delta\varphi)^2. \tag{3.1}$$

If the  $\rho$  fields have charge  $+2$  then  $D$  is positive definite and cannot vanish. This clearly provides an extra lifting term of order  $(T + T^*)^{-3}$  to the scalar potential precisely as proposed in [1]. This option was recently studied in a supergravity model motivated by string theory in [10].

Let us now consider what is probably the most generic case of a  $\rho$  field with charge  $-2$ . In this case the  $\varphi$  fields are massive and can be integrated out and we are only left with a tachyonic field  $\rho$ . This can be done since the  $\varphi$  fields have charge  $+1$  (same sign as the FI term). Also there is a mass term in the superpotential of the form  $\rho \varphi \tilde{\varphi}$  which gives a mass to  $\varphi$  once the field  $\rho$  of charge  $q = -2$  gets a vev. If the mass of  $\varphi$  is greater than the effective value of the renormalisation group invariant scale  $\Lambda$  of the non-abelian gauge theory, the fields  $\varphi$  can be integrated out and we are left with an effective theory in terms of only  $\rho$  and the Kähler modulus  $T$ .

We start then with the 4D supersymmetric effective action in terms of one matter field  $\rho$  with charge  $q = -2$  and one Kähler modulus  $T$  with an anomaly induced charge  $Q/(4\pi^2)$ . Before eliminating the  $\varphi$  fields, the Affleck-Dine-Seiberg superpotential has the form

$$W_{np} = \left( \frac{\Lambda^{3N_c - N_f}}{\det(\varphi\tilde{\varphi})} \right)^{\frac{1}{N_c - N_f}} \tag{3.2}$$

---

<sup>12</sup>In general  $\rho$  is a  $N_f \times N_f$  matrix in flavour space but for simplicity we will first restrict to a single field  $\rho$ , valid strictly for  $N_f = 1$  but capturing the main physics for the general case (we will discuss later the possible implications of the other fields  $\rho$  in the  $N_f \neq 1$  cases)

where  $\Lambda^{3N_c - N_f} = e^{-8\pi^2 T}$  is the scale of gaugino condensation,  $N_c$  is the number of colours determined by the number of un-magnetised D7 branes and  $N_f$  the number of flavours of chiral fields  $\varphi$ . This is gauge invariant if  $Q = -N_f$ .

After integrating out the  $\varphi$  fields, the Lagrangian is determined by the following superpotential and Kähler potential<sup>13</sup>

$$W = W_0 + \rho^a e^{-bT} \tag{3.3}$$

$$K = -2\log(\tau^{3/2} + \xi) + \frac{|\rho|^2}{\tau^{2/3}}, \tag{3.4}$$

where  $W_0$  is the constant flux superpotential,  $a, b$  are constants determined by the requirement that  $W$  has dimension three and is gauge invariant. So:

$$a = \frac{N_f}{N_c} \quad b = \frac{8\pi^2}{N_c}. \tag{3.5}$$

We have also defined  $2\tau = T + T^*$  and added the  $\alpha'$  correction to the Kähler potential specified by  $\xi$ . In (3.4) we have used the recent result in [28] regarding the moduli dependence of the matter field Kähler potential indicating a ‘modular weight’ of  $-2/3$  for the matter field  $\rho$ .

The scalar potential as a function of  $T$  and  $\rho$  is of the form:

$$V = V_F + V_D \tag{3.6}$$

with

$$V_F = e^K (D_i W \overline{D_j W} K^{i\bar{j}} - 3|W|^2), \quad V_D = \frac{g^2}{2} D^2 = \frac{1}{2\tau} \left( \frac{3Q}{8\pi^2\tau} - \frac{q|\rho|^2}{\tau^{2/3}} \right)^2 \tag{3.7}$$

by neglecting both the matter contribution to  $\partial_T K$  in  $D$ , which goes as  $-\frac{Q|\rho|^2}{12\pi^2\tau^{5/3}}$ , and the  $\alpha'$  correction parametrised by  $\xi$ . In principle we need to find the extrema of this potential for the two complex (four real) fields  $T, \rho$ . For the axionic parts  $\theta_T = \text{Im}T$  and  $\theta_\rho = \text{Arg}\rho$ , we can see that they only appear in the scalar potential through the non-perturbative part of  $W$  and therefore only in the combination  $\Theta = a\theta_\rho - b\theta_T$ . The orthogonal combination does not appear in the potential at all but it is just as well, since this is precisely the combination that is eaten by the anomalous U(1) gauge field to get a mass as it can be easily verified. Extremising with respect to  $\Theta$  is straightforward (since it only appears in the scalar potential through  $\cos \Theta$ , with extrema at  $\Theta = m\pi$ ). Therefore the relevant fields to concentrate on are  $\tau$  and the modulus of  $\rho$ .

Since

$$D_\rho W = a\rho^{a-1} e^{-bT} + \rho^* \tau^{-2/3} W \tag{3.8}$$

$$D_T W = -b\rho^a e^{-bT} - \left[ \frac{3}{2(\tau + \xi\tau^{-1/2})} + \frac{|\rho|^2}{3\tau^{5/3}} \right] W \tag{3.9}$$

---

<sup>13</sup> $W_0$  is the standard flux superpotential.  $\xi$  is the effect of the  $\alpha'$  corrections as computed in [12], that is proportional to the Euler number of the Calabi-Yau.

we can easily see that

$$q\rho\frac{D_\rho W}{W} + \frac{Q}{4\pi^2}\frac{D_T W}{W} = D. \tag{3.10}$$

Therefore, for a KKLT like scenario in which  $W_0$  is very small and there are solutions to  $D_T W = D_\rho W = 0$ , it can be seen that  $D = 0$  automatically. In practise it is easier to solve for  $\rho$  in  $D = 0$  and substitute in  $D_T W = 0$  to solve for  $\tau$ . This immediately reduces this system to the KKLT one with no relevant effect from the D-term. This is as expected from the discussions by Choi et al. [8]. We thus conclude that it is safe to ignore the effects of anomalous U(1)'s in the KKLT scenario, where the original AdS vacuum is supersymmetric. In this case both F and D terms vanish and we still have a supersymmetric AdS vacuum.

We will now consider a scenario similar to that of [15], with a one Kähler modulus Calabi-Yau manifold, and turn on RR and NS fluxes such that the flux-induced superpotential is  $W_0 \sim 1$  which is more generic. Therefore the nonperturbative effect stabilising the Kähler modulus  $T$  is much smaller in magnitude than  $W_0$ . Then the supersymmetry preserving condition  $D_T W = 0$  cannot be satisfied and there is no KKLT minimum. In a large  $\tau$  approximation, assuming that  $W_0$  dominates over the nonperturbative terms in  $V_F$ , and after minimising the phase of  $\rho$  and the imaginary part of  $T$ , the expression for  $V_F$  becomes:

$$V_F \sim \frac{|W_0|^2|\rho|^2}{3\tau^{11/3}} \left[ 1 + \frac{2|\rho|^2}{3\tau^{2/3}} \right] + \frac{\xi|W_0|^2}{\tau^{9/2}} + V_{\text{nonpert}} \tag{3.11}$$

Where  $V_{\text{nonpert}}$  is the non-perturbative part of  $V_F$ .

In order to minimise the full scalar potential with respect to  $\rho$ , we observe that for large  $\tau$  the D-term dominates and the minimum takes the form:

$$|\rho|^2 = \frac{3Q}{8\pi^2 q \tau^{1/3}} (1 + \epsilon), \tag{3.12}$$

with  $\epsilon \sim \frac{1}{\tau}$ . Substituting this result in  $V_D$  gives

$$V_D = \frac{9Q^2\epsilon^2}{2(8\pi^2)^2 q^2 \tau^3} \sim O(\tau^{-5}), \tag{3.13}$$

and  $V_F$  behaves as

$$V_F \sim |W_0|^2 \frac{Q}{8\pi^2 q \tau^4} + \frac{\xi|W_0|^2}{\tau^{9/2}} + V_{\text{nonpert}} \tag{3.14}$$

Where

$$V_{\text{nonpert}} = Ae^{-2b\tau} - BW_0 e^{-b\tau} \tag{3.15}$$

and  $A$  and  $B$  are functions of inverse powers of  $\tau$ . At leading order they are

$$\begin{aligned} A &= \frac{b^2 Q^a}{q^a (\frac{\tau}{2})^{1+a/3}} + \frac{Q^a (6b - 4/3ab + 3a^2 \frac{q}{Q})}{q^a (\frac{\tau}{2})^{2+a/3}} \\ B &= \frac{6bQ^{a/2}}{q^{a/2} (\frac{\tau}{2})^{2+a/6}} + \frac{aQ^{a/2}}{q^{a/2} (\frac{\tau}{2})^{3+a/6}}. \end{aligned} \tag{3.16}$$

Notice that the second and third term of expression (3.14) are as in the standard KKLT scenarios with added  $\alpha'$  corrections [15]. The net effect of the  $\rho$  field is adding the first

term (which dominates over the D-term at large  $\tau$ ). This term is precisely what we need in order to lift to de Sitter space. If without including this term the minimum for  $\tau$  is an AdS one, the potential at that minimum scales as  $-1/\tau^{9/2}$ . The first term in (3.14) dominates over this at large  $\tau$  since it scales as  $+1/\tau^4$ . Furthermore, since both powers are similar we need only a tuning of order  $1/\tau^{1/2}$  in the coefficient of the  $\rho$  induced  $V_F$  in order to have a de Sitter minimum (and not wash away completely the original AdS minimum). The de Sitter minimum can be obtained for values of  $\tau \sim 10 - 100$ . The necessary tuning is smaller than in the original KKLT case due to the higher power of the lifting term. Still the magnitude of this term can be controlled if the 2-cycle where the magnetic flux is at the tip of a warped throat as in the KKLT case. Notice that in this case the parameter  $Q$  would be modified by the warp factor as expected [1].

This is therefore an explicit realisation of de Sitter lifting from D-terms as proposed in [1]. Notice that both D and F terms are non-vanishing in the resulting minimum. Notice however that in the original discussion of [15] there were also de Sitter minima, even though at relatively small volume. The D-term lifting does not appear to be particularly useful in this case. Things are different in the more generic, many Kähler moduli case.

### 3.2 Two moduli case

Let us consider the toy model of a hyper-surface in the weighted projective space  $\mathbb{P}_{[1,1,1,6,9]}^4$  studied in [17, 29], with two Kähler moduli  $T_b$  and  $T_s$ .

The Kähler potential for the Kähler moduli is given by

$$K = -2 \log(\mathcal{V} + \xi) \quad \text{with} \quad \mathcal{V} = (T_b + T_b^*)^{3/2} - (T_s + T_s^*)^{3/2}. \quad (3.17)$$

We assume that a stack of D7 branes wraps the small 4-cycle, such that a nonperturbative superpotential is generated in the four dimensional effective field theory. This will generically result in stabilising the modulus  $T_b$  corresponding to the overall volume perturbatively at an exponentially large value while  $T_s$  will be fixed at an  $\mathcal{O}(1)$  value. For details of the construction we refer to [16].

#### 3.2.1 Fluxes on the large cycle

Let us turn on magnetic flux on a 2-cycle which is a sub-cycle of the large 4-cycle corresponding to  $T_b$ .

For simplicity we will assume that the modulus  $T_b$  is entirely perturbatively stabilised, so that it does not appear in the superpotential (this is justified a posteriori if the volume is exponentially large we can neglect the nonperturbative dependence on  $T_b$ ). This will be the case as long as there are a sufficient number of massless adjoint multiplets living on the world-volume of the branes left after turning on RR and NS fluxes. This means that the superpotential can be written as

$$W = W_0 + A_s e^{-a_s T_s}. \quad (3.18)$$

We are assuming that  $T_s$  is not charged under the  $U(1)$  living on branes wrapping the large cycle (which will be the case if the 4-cycles corresponding to  $T_s$  and  $T_b$  do not intersect



over any 2-cycle), so that we do not need to include open string fields in the superpotential formula (3.18).

As explained in [16], the nonperturbative effects do not destabilise the flux-stabilised complex structure and dilaton moduli. Therefore we will only discuss the Kähler moduli dependence of the scalar potential. For the time being, we assume an arbitrary parametrisation for the Kähler potential of the  $\rho$  field:

$$K = -2 \log(\mathcal{V}) + c \frac{\rho \rho^*}{(T_b + T_b^*)^\alpha}. \quad (3.19)$$

It is easy to see that under a redefinition of  $\rho$ ,  $\rho \rightarrow \rho/\sqrt{c}$ , the dependence on  $c$  is eliminated both from the Kähler potential and from the D-term. Therefore we may set  $c = 1$ .

The metric on moduli space is computed in appendix B. It can be used to compute the full scalar potential, in particular the lowest order F-term contribution involving  $|\rho|^2$  which will be of interest to us.

Let us investigate where  $\rho$  is stabilised. If there were no F-term contributions to its potential,  $\rho$  would be stabilised by the requirement that the D-term vanish. However, there are F-term contributions coming from  $K^{\rho\bar{\rho}}(D_\rho W)(\overline{D_\rho W})$ , as well as  $K^{b\bar{b}}(D_b W)(\overline{D_b W})$  and the mixed terms  $K^{i\bar{i}}(D_i W)(\overline{D_i W})$  (and complex conjugates). All these contributions turn out to scale as the same power of  $1/\mathcal{V}$ :

$$\frac{1}{\mathcal{V}^{2+\frac{2}{3}\alpha}} |\rho|^2 |W_0|^2. \quad (3.20)$$

The coefficient of this term can be shown to be  $(1 - \alpha)$ . This is done in appendix C. As discussed earlier, the sign of the charge of the  $\varphi$  fields is aligned with the sign of the FI term, so they always have a positive mass, while the charges of  $\varphi$  and  $\rho$  can be either of the same or opposite sign. Let us first assume that the sign of the charge of the  $\varphi$ 's is the same as the charge of  $\rho$ .

Therefore the D-term now has the form

$$V_D = \frac{1}{T_b + T_b^*} \left( \frac{p}{T_b + T_b^*} + k \sum_i \frac{|\rho_i|^2}{(T_b + T_b^*)^\alpha} + l \sum_j \frac{|\varphi_j|^2}{(T_b + T_b^*)^\alpha} \right)^2, \quad (3.21)$$

There are also F-term contributions to the scalar potential involving  $\rho_i$  and  $\varphi_j$ , of the form

$$\sum_i \frac{|\rho_i|^2}{(T_b + T_b^*)^{3+\alpha}} + \sum_j \frac{|\varphi_j|^2}{(T_b + T_b^*)^{3+\alpha}}. \quad (3.22)$$

The full potential is clearly minimised for  $\varphi = \rho = 0$ , which gives an uplift potential of the form

$$V_D = \frac{p^2}{(T_b + T_b^*)^3} \sim \frac{p^2}{\mathcal{V}^2}. \quad (3.23)$$

This will be sufficient to uplift a nonsupersymmetric minimum with cosmological constant  $\sim -1/\mathcal{V}^3$  as long as there is a fine tuning in  $\epsilon = p^2$  of order  $1/\mathcal{V}$ . Recalling that  $p = Q/(4\pi^2)$ , with  $Q$  an integer, we can see that  $p^2$  is naturally of order  $1/1000$ , so for volumes in the

region  $\mathcal{V} \sim 10^3$  no extra fine tuning (coming from warping or other mechanisms) is required. Note that this is a realisation of the original proposal of [1].

In this analysis we did not include the non-abelian D-term for the quark fields,

$$V_D^{\text{nonab}} = \frac{1}{\text{Ref}} \sum_a ((\partial_{\varphi_i} K) T_{ij}^a \varphi_j)^2, \tag{3.24}$$

with  $T^a$  the generators of the nonabelian gauge group.  $V_D^{\text{nonab}}$  is a positive definite contribution to the energy which only depends on the  $\varphi$  fields, so  $\varphi = 0$  is still a minimum.

Suppose now that the charge of the  $\rho$  fields is opposite in sign to the charge of the  $\varphi$  fields. Then the scalar potential is minimised at  $\varphi = 0$  and only one of the  $\rho$  fields (say,  $\rho \equiv \rho_1$ ) is nonzero. That there is a stationary point with these properties is clear. That it is a minimum can also easily be shown, after observing that, after minimising,

$$D = \frac{p}{T_b + T_b^*} - k \frac{|\rho|^2}{(T_b + T_b^*)^\alpha} + l \frac{|\varphi|^2}{(T_b + T_b^*)^\alpha} = \frac{1}{2k(T_b + T_b^*)^2}. \tag{3.25}$$

Then one has

$$\frac{\partial^2 V}{\partial \varphi \partial \varphi^*} = \frac{1}{(T_b + T_b^*)^{3+\alpha}} + \frac{2l}{(T_b + T_b^*)^{\alpha+1}} D > 0. \tag{3.26}$$

The  $\rho_j, j > 1$  are flat directions of  $V$ .

The D-term contribution to the mass of  $\rho$  arises from

$$\frac{1}{T_b + T_b^*} \left( \frac{r}{(T_b + T_b^*)} - \frac{q|\rho|^2}{(T_b + T_b^*)^\alpha} \right)^2, \tag{3.27}$$

and scales as  $|\rho|^2/\mathcal{V}^{4/3+2\alpha/3}$ . Here any  $\mathcal{O}(1)$  constant factors from differentiating  $K$  have been absorbed into  $q, r > 0$  and the negative sign has been explicitly inserted to emphasise that  $\rho$  is tachyonic.

Thus the minimum for  $\rho$  will be such that the D-term is almost, but not completely, cancelled. More explicitly, let us consider the  $|\rho|^2$  contributions from the F- and D-terms:

$$V = f(T_s, T_b) + \frac{(1-\alpha)}{\mathcal{V}^{2+\frac{2}{3}\alpha}} |\rho|^2 |W_0|^2 + \frac{1}{T_b + T_b^*} \left( \frac{r}{(T_b + T_b^*)} - q \frac{|\rho|^2}{(T_b + T_b^*)^\alpha} \right)^2. \tag{3.28}$$

Minimising with respect to  $\rho$  we find

$$\frac{r}{(T_b + T_b^*)} - \frac{q|\rho|^2}{(T_b + T_b^*)^\alpha} = \frac{(1-\alpha)}{2q} \frac{1}{\mathcal{V}^{\frac{4}{3}}} |W_0|^2, \tag{3.29}$$

so that

$$|\rho|^2 \sim \mathcal{V}^{2(\alpha-1)/3}, \tag{3.30}$$

and the F-term contribution to the scalar potential,  $\frac{(1-\alpha)}{\mathcal{V}^{2+\frac{2}{3}\alpha}} |\rho|^2 |W_0|^2$ , scales as  $\mathcal{V}^{-8/3}$ , while the D-term contribution scales as  $\mathcal{V}^{-10/3}$ .

Let us now investigate the resulting scalar potential, setting  $\alpha = 2/3$  as explained in section 3.1., and considering only lowest order terms in the  $1/\mathcal{V}$  expansion<sup>14</sup>

$$V = \frac{\lambda\sqrt{\tau_s}e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{\mu}{\mathcal{V}^2}\tau_s e^{-a_s\tau_s} + \frac{\nu}{\mathcal{V}^{8/3}} + \frac{\xi}{\mathcal{V}^3}. \tag{3.31}$$

It turns out that neglecting the leading order  $\alpha'$ -correction, which scales as  $\xi/\mathcal{V}^3$ , the potential has no minima. There is a stationary point determined by

$$e^{a_s\tau_s} = \left(\frac{3^6}{8^3 \cdot 2^5}\right) \left(\frac{a_s A_s}{W_0}\right) \tau_s^4 \tag{3.32}$$

$$\mathcal{V} = \frac{3^6}{2^{15}} \sqrt{\tau_s} \tau_s^4, \tag{3.33}$$

which can numerically be checked to be a saddle point. Therefore we need to include the  $\alpha'$  corrections.

We know that including the  $\alpha'$  corrections to the scalar potential gives rise to non-supersymmetric AdS minima before any lifting allows a de Sitter lifting. One must then add the lift in a controlled manner to avoid wiping out the minimum altogether. This amounts to assuming a fine tuning of the coefficient of the uplifting term with respect to the coefficient of the  $\alpha'$  correction term as in the original KKLT scenario. Given that the cosmological constant of the AdS minimum is roughly  $-1/\mathcal{V}^3$ , and that the lifting term is of the form  $1/\mathcal{V}^{8/3}$ , it can be seen that a fine tuning of order  $1/\mathcal{V}^{1/3}$  is required.

We checked numerically that lifting to a stable de Sitter vacuum can indeed be achieved, with values  $a_s = 2, \lambda = 4, \mu = 20, \xi = 131, \nu = 3.125$ . The resulting volume is roughly 50000, with  $\mathcal{V}^{-1/3} \approx 0.06$  and  $\nu/\xi \approx 0.02$ .

To achieve a fine tuning of this magnitude, a hierarchically small FI term is required. This may be achieved already due to the suppression factor of  $4\pi^2$  in the FI term. It may also be the result of warping: the low energy effective action for the magnetised D7 brane contains a term coming from the Yang-Mills kinetic terms, of the form

$$\int d^8x \sqrt{\det g_8} F_{mn} F^{mn}, \tag{3.34}$$

where  $g_8$  is the induced metric on the D7 brane. Assuming a warped ansatz for the metric of the form

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} g_{mn}(y) dy^m dy^n, \tag{3.35}$$

we observe that the powers of  $e^{2A(y)}$  cancel out of  $\sqrt{\det g_8}$ . Assuming that the D7 brane has a constant warping  $A(y) = A$  over its worldvolume, the raising of indices in  $F^{mn}$  gives  $e^{4A}$ , indicating that the term (3.34) is suppressed by an overall warp factor.

This effect may be hard to achieve in practise since the 4-modulus  $T_b$  corresponds to the overall volume of the Calabi-Yau, and it is difficult to envisage a situation in which most of that 4-cycle can be in a highly warped region.

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<sup>14</sup>The constants  $\mu, \nu$  and  $\xi$  are defined as follows  $\mu \sim a_s |A_s W_0|, \lambda \sim a_s^2 |A_s|^2, \nu \sim |W_0|^2, \xi \sim -\chi(M) |W_0|^2$ , with  $\chi(M)$  the Euler number of the Calabi-Yau. See [16] for details.

### 3.2.2 Fluxes on the small cycle

Let us now consider turning on magnetic flux on a brane wrapping one of the small 4-cycles. We need to include the dependence of the Kähler metrics on the small modulus. Following [28], we parametrise this dependence as

$$K = -2 \log \mathcal{V} + \frac{|\rho|^2}{(T_s + T_s^*)^\beta \mathcal{V}^\alpha}, \quad (3.36)$$

where  $T_s$  is the modulus of the small 4-cycle. Let us estimate first the contribution to the F-term energy, coming from  $e^K K^{\rho\rho} |D_\rho W|^2$ . This can be seen to scale as

$$|\rho|^2 |W_0|^2 \frac{1}{\mathcal{V}^{2+\alpha} (T_s + T_s^*)^\beta}. \quad (3.37)$$

In fact, a careful analysis similar to the one in appendix C shows that the F-term contribution is

$$\left(1 - \frac{3\alpha}{2} + 8\beta\right) |\rho|^2 |W_0|^2 \frac{1}{\mathcal{V}^{2+\alpha} (T_s + T_s^*)^\beta}. \quad (3.38)$$

If  $\beta$  is negative, and larger than  $1/8$  in magnitude, the overall coefficient in (3.38) will be negative.

The superpotential for this case may be written as

$$W = W_0 + \rho\varphi\tilde{\varphi} + A_s \frac{e^{-a_s T_s}}{(\det(\varphi\tilde{\varphi}))^p} \quad (3.39)$$

with  $p$  the corresponding power in the Affleck-Dine-Seiberg superpotential. Note that we do not include a term allowed by gauge invariance,  $\rho^a e^{-bT_s}$ , since the  $\rho$  fields are not charged under the nonabelian gauge group which condenses and hence are not expected to appear in the superpotential (note that once we integrate out the  $\varphi$ 's such coupling will appear).

As mentioned in section 2.3, there are two possibilities for which of the Kähler moduli ( $T_b$  and  $T_s$ ) becomes charged under the anomalous U(1). Let us first consider the case that  $T_s$  becomes charged. The charges of the  $\varphi$  fields relative to the charge of  $T_s$  under the anomalous U(1) are fixed by the gauge invariance of (3.39). The sign of the FI term may or may not change from the case of fluxes on the large cycle, depending on whether  $(\partial_{T_s} K)$  has the same or opposite sign to  $(\partial_{T_b} K)$ .

In the case that the  $\varphi$  fields have positive mass squared, we can integrate them out keeping only  $\rho$ . Then if  $\rho$  has positive mass squared, the lifting can be achieved since the D-term scales as  $1/\mathcal{V}^2$ . If however,  $\rho$  is tachyonic, then after minimising with respect to  $\rho$ , the combination of (3.38) and the D-term potential gives a negative energy contribution, not allowing for a de Sitter minimum.

In the large  $\mathcal{V}$  limit, the D-term potential is

$$\frac{1}{T_s + T_s^*} \left( \frac{r}{\mathcal{V}} - \frac{|\rho|^2 q}{\mathcal{V}^\alpha (T_s + T_s^*)^\beta} \right)^2 \quad (3.40)$$

Minimising the sum of F-term and D-term contributions to the energy with respect to  $\rho$  gives

$$\left(1 - \frac{3\alpha}{2} + 8\beta\right) \cdot \frac{|W_0|^2}{\mathcal{V}^{2+\alpha}(T_s + T_s^*)^\beta} = \frac{2q}{\mathcal{V}^\alpha(T_s + T_s^*)^{\beta+1}} D, \quad (3.41)$$

with

$$D \equiv \left(\frac{r}{\mathcal{V}} - \frac{|\rho|^2 q}{\mathcal{V}^\alpha(T_s + T_s^*)^\beta}\right) \quad (3.42)$$

In the large volume limit one has  $D \sim 1/\mathcal{V}^2$ . Therefore, the D-term potential behaves like  $D^2 \sim 1/\mathcal{V}^4$ .

The VEV of  $\rho$  is fixed by

$$|\rho|^2 \sim \frac{r\mathcal{V}^\alpha}{q\mathcal{V}}. \quad (3.43)$$

The F-term contribution to the energy is therefore  $\sim 1/\mathcal{V}^3$  and is insufficient to lift the vacuum to de Sitter, even if its overall coefficient is positive. Interestingly, this behaves like  $\alpha'$  corrections to the scalar potential and could be used for proving the existence of large volume minima for Calabi-Yau manifolds  $M$  with  $\chi(M) = 2(h^{1,1} - h^{1,2}) > 0$ . This is a very model dependent statement, though, since one requires  $r/q$  to be large enough.

If the  $\varphi$  fields have opposite charge to that of the FI term, they cannot be integrated out, and since they can cancel the D-term, a similar argument to the one above applies so that lifting cannot be achieved.

The following case is that the field that becomes charged under the anomalous  $U(1)$  is  $T_b$  rather than  $T_s$ . Anomaly cancellation implies then that there cannot be  $\varphi, \tilde{\varphi}$  fields living between magnetised and unmagnetised branes wrapping the small 4-cycle. The superpotential is thus given by (3.18). Since the sign of the FI term is determined by  $\partial_b K$ , it is negative. In case there are branes wrapping the large cycle, an anomaly is generated, and consequently  $\varphi$  fields must exist (this time corresponding to strings stretched between the branes on the small 4-cycle and those wrapping the large 4-cycle) and will have a positive mass. After integrating them out, the D-term potential is of the form

$$\frac{1}{T_s + T_s^*} \left(\frac{r}{T_b + T_b^*} \pm \frac{q|\rho|^2}{\mathcal{V}^\alpha(T_s + T_s^*)^\beta}\right)^2 \quad (3.44)$$

If the sign of  $\rho$  is positive, lifting can be achieved with an appropriate amount of warping. If it is negative, and the sign of the F-term contribution (determined by  $1 - 3\alpha/2 + 8\beta$ ) is negative, minimising with respect to  $\rho$  will only yield a negative total contribution. Notice that this will likely be the case if  $\alpha > 0$  and  $\beta < 0$ , as advocated in [28]. In this case, lifting would be impossible. However, the result of [28] relies on the assumption that the matter fields localised in the small cycle decouple from the dynamics of the large cycle, which will not generically be the case if the two cycles intersect and there are branes wrapping both of them.

The last case we will analyse is having magnetised branes wrapping the large cycle in such a way that it is the small cycle the one that gets charged. This case is rather similar to the one with fluxes on the small cycle with the small cycle becoming charged, and, similarly to that case, it does not give de Sitter vacua.

## 4. Conclusions

We have studied the spectrum of chiral fields in magnetised D7 branes, with a gauge group  $SU(N_c) \times U(1)$  and fields  $\varphi, \tilde{\varphi}$  in the fundamental of  $SU(N_c)$  and anomalous  $U(1)$  charge  $+1$ , and  $SU(N_c)$  singlets  $\rho$  with anomalous  $U(1)$  charge  $\pm 2$ . This has allowed us to consider several scenarios depending on the number of moduli and the location of the D7 branes and magnetic fluxes.

- (i) If all matter fields have positive charge, then the D-term cannot vanish and the minimum of the scalar potential can be lifted to de Sitter space. This can be considered a string realisation of the model considered by Achúcarro et al. [10].
- (ii) The fields  $\varphi, \tilde{\varphi}$  have positive charge but the fields  $\rho$  have negative charge. In this case the KKLT-like scenario with supersymmetric AdS minimum remains essentially unchanged and the D-term does not lift the minimum to de Sitter space, in full agreement with the arguments of Choi et al. However once  $\alpha'$  corrections are included both F- and D-terms are non-vanishing and the minimum may or may not be lifted to de Sitter depending on the type of 4-cycle that the D7 branes wrap.

The lifting mechanism works naturally for volumes of order in the thousands, but in order to work properly for larger volumes, the 2-cycle in which the magnetic flux is turned on has to be at the tip of a deeply warped throat. The lifting is achieved by tuning the warp factor appropriately as in the original KKLT scenario. The necessary geometry is possible to realise<sup>15</sup> but an explicit construction is beyond the scope of this article.

In the cases where the D-term lifting does not happen, a relevant question would be if the D-terms change the nature of the minima found in the absence of magnetic fluxes. We have seen that, as expected, the supersymmetric AdS minima such as the original KKLT scenario are not affected by the anomalous  $U(1)$ . Furthermore the same happens for large volume minima. The fact that there are minima with exponentially large volume remains true even after adding the D-terms, independent of the fact that they lift to de Sitter space or not. This makes these scenarios more robust.

Our mechanism of moduli stabilisation can be seen as a generalisation of the mechanism proposed in [13] of using only D-terms and soft supersymmetry breaking terms to stabilise the Kähler moduli. In principle we could achieve this by turning-off the non-perturbative effects.<sup>16</sup> However when we turn them off, the field equations for the Kähler moduli become linear and they are not stabilised. Introduction of (anti) D9-branes may be needed to achieve stabilisation but then care must be taken about avoiding Freed-Witten anomalies. This is clearly model dependent and will not be addressed further here.

We would like to point out that a crucial part of our calculations was to use the recently computed Kähler moduli dependence of matter fields [28]. It was crucial to know the modular weights of the matter fields in order to establish the positivity of the contribution

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<sup>15</sup>See for instance [30]. We thank H. Verlinde for discussions on this point.

<sup>16</sup>A related mechanism of Kahler moduli stabilisation with magnetic fluxes in toroidal setups has been proposed in [14].

of the matter fields to the F-term part of the potential. Here we used the simplest case in which all matter fields are assumed to wrap the same cycle. Other cases discussed in [28] are easily incorporated.

In most of our considerations we only included a single matter field  $\rho$  singlet under the non-abelian gauge symmetry. In general it is a matrix of these fields  $\rho_{ij}$  with  $i, j$  flavour indices. In the nonperturbative superpotential, the term  $\rho^{N_F}$  means actually  $\det \rho_{ij}$  and in the D-terms and Kähler potential, these fields appear in the combination  $\sum_{ij} |\rho_{ij}|^2$ . This means that there are several combinations of these fields that do not appear explicitly in the potential and could remain flat. It would be interesting to study their possible role as inflaton candidates once further corrections to the potential are included. In any case there is no problem about their stabilisation since they are bounded complex quantities which are always stabilised at finite values.

In summary, we have generalised the current discussions on moduli stabilisation to include magnetised D7 branes of the type expected to include the standard model in a fully realistic setting. We hope this is only a first step towards a more stringy realisation of realistic chiral models within the KKLT and exponentially large volume scenarios. The fact that the D-terms can be actually used for de Sitter lifting is very encouraging. Open questions regarding the actual structure of soft supersymmetry breaking terms remain to be discussed in detail. In particular, the F-term of the matter field  $\rho$  is generally non-zero and could contribute to the structure of soft supersymmetry breaking in the observable sector. Here we can only say that, as already discussed in the literature, the lifting mechanism is the leading source of supersymmetry breaking in KKLT models [8, 31] but its effect on the exponentially large volumes is less relevant due to the fact that the original AdS minimum is already non-supersymmetric [17, 32]<sup>17</sup>.

With this work we believe to have clarified a number of important issues regarding the effects of anomalous U(1)'s on the moduli stabilisation procedure. This puts the mechanisms of moduli stabilisation on firmer grounds and also allows the possibility for de Sitter lifting in a controllable manner. The explicit realisation of the lifting mechanism is model dependent, which we have illustrated with some representative cases. We hope these techniques and results will be useful in detailed constructions of realistic string models including moduli stabilisation.

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<sup>17</sup>Notice that in all cases supersymmetry is broken after the lifting. A TeV gravitino mass can be obtained naturally in the large volume scenario since  $m_{3/2} \sim W_0 M_P / \mathcal{V}$  whereas in the KKLT scenario a fine tuning of  $W_0$  will still be needed to get small enough gravitino masses.

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## A. Anomaly cancellation

We check in this section<sup>18</sup> how the matter content described in section 2.3, together with the GS mechanism, cancels all possible anomalies that could be present in our setup.

### A.1 $U(1)_a - SU(N_b)^2$ anomalies

The mixed anomaly is given by the formula

$$\mathcal{A}_{ab} = \sum_r Q_a(r) \cdot C_b(r) \quad (\text{A.1})$$

where  $C_b(r)$  is the quadratic Casimir of  $SU(N_b)$ , and  $Q_a(r)$  is the  $U(1)$  charge. Let us consider first the case  $a \neq b$ . The only fields relevant for this computation are the  $\varphi$  and  $\tilde{\varphi}$ . Knowing that for both fundamental and anti-fundamental representations one has  $C(r) = \frac{1}{2}$ , one readily gets

$$\mathcal{A}_{ab} = \frac{1}{2} N_a (I_{ab} + I_{ab'}). \quad (\text{A.2})$$

This anomaly should eventually be cancelled by a GS term. Now, consider the case  $a = b$ . Now, the relevant fields are not only the  $\varphi$ 's and  $\tilde{\varphi}$ 's but also the symmetric and the antisymmetric. Knowing that

$$C(\square_a) = \frac{N_a + 2}{2}, \quad (\text{A.3})$$

$$C(\square_a) = \frac{N_a - 2}{2}, \quad (\text{A.4})$$

$$Q(\square_a) = Q(\square_{N_a}) = +2, \quad (\text{A.5})$$

we get

$$\begin{aligned} \mathcal{A}_{aa} &= \frac{1}{2} \sum_{c \neq a} N_c (I_{ac} + I_{ac'}) + 2 \cdot \frac{N_a + 2}{2} (\#\square) + 2 \cdot \frac{N_a - 2}{2} (\#\square) \\ &= \frac{1}{2b} \left[ b N_a I_{aa'} + b \sum_{c \neq a} N_c (I_{ac} + I_{ac'}) - I_{aO} \right] + \frac{1}{2} N_a I_{aa'}. \end{aligned} \quad (\text{A.6})$$

Now, the term between square brackets vanishes because of (2.12), and the remaining anomaly is just  $\frac{1}{2} N_a I_{aa'}$ . It has the same form as (A.2), so we can say in full generality

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<sup>18</sup>This section follows closely refs. [25, 33]. Though these references dealt only with toroidal and orbifold constructions, their results can be extended to more general constructions since they rely only in the bilinearity of the intersection product between D7 and D5 charges.



that the leftover anomaly  $U(1)_a - SU(N_b)$  anomaly between some  $a$  and  $b$  sectors is just given by

$$\mathcal{A}_{ab} = \frac{1}{2} N_a (I_{ab} + I_{ab'}). \quad (\text{A.7})$$

This is the term that must be cancelled by the GS mechanism.<sup>19</sup> It is easy to see that it is indeed the case. As explained in the main text, magnetic flux in the world-volume of the D-brane  $a$  generates a charge for every modulus  $T_i$  which intersects the two-cycle(s) where magnetic field has been turned on. Wrapping a D7-brane in the four cycle associated to  $T_i$  induces an anomaly for the  $U(1)_a$ . The anomaly is given precisely by the products of the coefficients of  $\int_{M^4} D_{2,i} \wedge F_a$  and  $\int_{M^4} \text{Im} T_i F_i \wedge F_i$ , with  $D_{2,i}$  being the Hodge dual in four dimensions of  $\text{Im} T_i$ , and  $F_i$  the  $SU(N_i)$  field strengths. But this coefficient is precisely of the form (A.7), since it is equal to the D5 charge of the brane  $a$  times the D7 charge of the brane  $i$ . Note that the factor  $N_a$  arises in the CS term from the normalisation of the  $U(1)$ .

## A.2 $U(1)_a - U(1)_b^2$ anomalies

Let us analyse the case of the cubic anomaly. Let us start by assuming  $a \neq b$ . This anomaly is given by

$$\mathcal{A}_{ab} = \sum_r Q_a(r) \cdot Q_b^2(r). \quad (\text{A.8})$$

In this case only the  $\varphi$  and  $\tilde{\varphi}$  fields are relevant and one gets

$$\mathcal{A}_{ab} = N_a N_b (I_{ab} + I_{ab'}) \quad (\text{A.9})$$

The factor  $N_a N_b$  comes since in a bifundamental representation there are  $N_a N_b$  fields from the  $U(1)$  point of view. In the case  $a = b$ , we have to take into account that there a number of  $\rho$  fields given by (2.24) with charge  $+2$  (again, in case the number is negative, it means that the charge is  $-2$  so that everything is consistent). In this case the anomaly reads (the  $1/3$  factor comes from the symmetry of the diagram)

$$\begin{aligned} \mathcal{A}_{ab} &= \frac{1}{3} \left( N_a \sum_{c \neq a} N_c (I_{ac} + I_{ac'}) + 2^3 \left[ \frac{N_a^2}{2} I_{aa'} - \frac{N_a}{8b} I_{aO} \right] \right) \\ &= \frac{N_a}{3b} \left[ b N_a I_{aa'} + b \sum_{c \neq a} N_c (I_{ac} + I_{ac'}) - I_{aO} \right] + N_a^2 I_{aa'}. \end{aligned} \quad (\text{A.10})$$

Again, the term in square brackets vanish because of (2.12) and the remainder is  $N_a^2 I_{aa'}$ , whose form is the same as that of (A.9). We can write in full generality that the remainder of the cubic anomaly  $U(1)_a - U(1)_b^2$  is given by

$$\mathcal{A}_{ab} = N_a N_b (I_{ab} + I_{ab'}). \quad (\text{A.11})$$

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<sup>19</sup>We recommend the reference [34] for a nice and detailed explanation of how the GS mechanism takes place from the field theory point of view.

Again, this term will be cancelled by the GS mechanism. It is easy to see that this is the case, following a similar argument to that sketched for the mixed anomaly case. Note that in this case there is an extra factor of  $N_b$  appearing from the cubic case due to the fact that we are considering the coupling to the  $U(1)_b$  field, coming from a term  $\sim N_b \int_{M^4} F_b \wedge F_b$ ,  $F_b$  being in this case the  $U(1)$  field strengths.

### A.3 Gravitational anomalies

The gravitational anomaly for a given  $U(1)_a$  group is just given by

$$\mathcal{A}_a = \sum_r Q_a(r) = \text{Tr } Q_a. \tag{A.12}$$

In our case this is given by

$$\mathcal{A}_a = N_a \sum_{c \neq a} N_c (I_{ac} + I_{ac'}) + 2 \left( \frac{N_a^2}{2} I_{aa'} - \frac{N_a}{8b} I_{aO} \right). \tag{A.13}$$

After imposing the tadpole cancellation we obtain

$$\mathcal{A}_a = \frac{3}{4b} N_a I_{aO}. \tag{A.14}$$

This is the bit that has to be cancelled by the GS mechanism. One can check it does, following the steps outlined in [25].

## B. Metrics on moduli space

We now compute the metric and inverse metric on moduli space, in the limit of (the real part of)  $T_b$  large. The relationship between the volume and  $T_b$  is (for large  $T_b$ , and up to constant factors)  $\mathcal{V} \sim T_b^{3/2}$ .

$$K_{i\bar{j}} = \begin{pmatrix} 1/\mathcal{V}^{4/3} & 1/\mathcal{V}^{5/3} & 1/\mathcal{V}^{\frac{2}{3}(\alpha+1)} \\ 1/\mathcal{V}^{5/3} & 1/\mathcal{V} & 0 \\ 1/\mathcal{V}^{\frac{2}{3}(\alpha+1)} & 0 & 1/\mathcal{V}^{\frac{2}{3}\alpha} \end{pmatrix}. \tag{B.1}$$

Here the moduli are put in order  $\{1, 2, 3\} = \{T_b, T_s, \rho\}$ .

The determinant of  $K_{i\bar{j}}$  is  $\mathcal{V}^{-\frac{2}{3}\alpha - \frac{7}{3}}$ . The inverse metric is thus

$$K^{i\bar{j}} = \mathcal{V}^{\frac{2}{3}\alpha + \frac{7}{3}} \begin{pmatrix} \mathcal{V}^{-2/3\alpha-1} & \mathcal{V}^{-2/3\alpha-\frac{5}{3}} & \mathcal{V}^{-2/3\alpha-\frac{5}{3}} \\ \mathcal{V}^{-2/3\alpha-\frac{5}{3}} & \mathcal{V}^{-2/3\alpha-\frac{4}{3}} & \mathcal{V}^{-2/3\alpha-\frac{7}{3}} \\ \mathcal{V}^{-2/3\alpha-\frac{5}{3}} & \mathcal{V}^{-2/3\alpha-\frac{7}{3}} & \mathcal{V}^{-\frac{7}{3}} \end{pmatrix}. \tag{B.2}$$

So that

$$K^{i\bar{j}} = \begin{pmatrix} \mathcal{V}^{4/3} & \mathcal{V}^{2/3} & \mathcal{V}^{2/3} \\ \mathcal{V}^{2/3} & \mathcal{V} & 1 \\ \mathcal{V}^{2/3} & 1 & \mathcal{V}^{\frac{2}{3}\alpha} \end{pmatrix}. \tag{B.3}$$

Following [16], let us now consider the limit of large  $\mathcal{V}$  such that  $\mathcal{V} \sim e^{a_s \tau_s}$ . The first thing to consider is where the axion of  $T_s$  is fixed. The relevant terms, before including

the effects of  $\rho$ , are the mixed terms from  $K^{s\bar{b}}D_sW\overline{D_bW}$ , of the form  $K^{s\bar{b}}(\partial_bK)\overline{W}(\partial_sW)$ . This term scales as  $W(\partial_sW)$  with  $\mathcal{V}$ . After including the terms with  $\rho$ , there are more contributions to the axion potential, coming from the mixed terms in  $K^{s\bar{\rho}}(D_sW)\overline{(D_\rho W)}$ ,  $K^{b\bar{\rho}}(D_bW)\overline{(D_\rho W)}$ ,  $K^{\rho\bar{b}}(D_\rho W)\overline{(D_bW)}$ , and their complex conjugates. The parts of these terms involving  $\partial_\rho W$  are all vanishing as we are assuming there is no nonperturbative contribution involving  $T_{b,\rho}$  in the superpotential. Therefore, from the first term, only  $K^{s\bar{\rho}}(\partial_sW)\overline{(\partial_\rho K)W}$  is relevant, scaling as  $\mathcal{V}^{-\frac{2}{3}\alpha}(\partial_sW)W$ .

Thus the axion is still fixed mainly by the  $K^{s\bar{b}}(\partial_bK)\overline{W}(\partial_sW)$  term, at a value which makes the overall sign of this term negative.

### C. Computing the $|\rho|^2$ F-term contribution

The determinant of the metric  $K_{i\bar{j}}$  in (B.1) can be seen to scale as  $K_{bb}K_{ss}K_{\rho\rho} + K_{\rho b}K_{b\rho}K_{ss}$ . These terms can be estimated individually as

$$K_{b\bar{b}} = \frac{3}{\mathcal{V}^{4/3}} + \frac{\alpha(\alpha+1)|\rho|^2}{\mathcal{V}^{2(\alpha+2)/3}} \quad (\text{C.1})$$

$$K_{s\bar{s}} = \frac{3}{2\mathcal{V}(T_s + T_s^*)^{1/2}} \quad (\text{C.2})$$

$$K_{\rho\bar{\rho}} = \frac{1}{\mathcal{V}^{2\alpha/3}} \quad (\text{C.3})$$

$$K_{b\rho} = -\frac{\alpha\rho}{\mathcal{V}^{2(\alpha+1)/3}}. \quad (\text{C.4})$$

The first contribution which includes a  $|\rho|^2$  factor comes from the term in the scalar potential

$$K^{\rho\bar{\rho}}|(\partial_\rho K)W|^2, \quad (\text{C.5})$$

and the contribution turns out to be

$$\frac{|\rho|^2}{\mathcal{V}^{2\alpha/3}}|W|^2. \quad (\text{C.6})$$

The next contribution is from  $K^{\rho\bar{b}}(\partial_\rho K)W\overline{(\partial_b K)W}$ , and turns out to give

$$-\frac{\alpha|\rho|^2}{\mathcal{V}^{2\alpha/3}}. \quad (\text{C.7})$$

There is an identical contribution from the conjugate of this term.

The next contribution comes from  $K^{\rho\bar{s}}(\partial_\rho K)W\overline{(\partial_s W)W}$ , whose scaling with volume is

$$-\frac{3\alpha(T_s + T_s^*)^{3/2}|W|^2|\rho|^2}{\mathcal{V}^{2\alpha/3+13/3}}. \quad (\text{C.8})$$

The following term to consider is  $K^{b\bar{b}}(\partial_b K)W\overline{(\partial_b K)W}$  (there is a term in the inverse metric appearing in  $K^{b\bar{b}}$  containing  $|\rho|^2$ , and also two more terms coming from the  $(\partial_b K)$  factors). The scaling is given by

$$\frac{\alpha}{\mathcal{V}^{2\alpha/3}}|\rho|^2|W_0|^2 \quad (\text{C.9})$$

The next term is  $K^{s\bar{s}}(\partial_s K)W(\overline{\partial_s K})\overline{W}$ , giving only terms which scale as

$$\frac{1}{\mathcal{V}^{2\alpha/3+1/3}}, \tag{C.10}$$

which is suppressed with respect to the contributions we computed before.

Lastly, one may also consider  $K^{s\bar{b}}(\partial_s K)W(\overline{\partial_b K})\overline{W}$ . This gives

$$\frac{|\rho|^2}{\mathcal{V}^{2\alpha/3+2/3}}. \tag{C.11}$$

In total, after taking into account the pre-factor of  $e^K \sim 1/\mathcal{V}^2$ , the highest order F-term contribution including a  $|\rho|^2$  factor is

$$(1 - \alpha) \frac{|\rho|^2 |W_0|^2}{\mathcal{V}^{2\alpha/3}}. \tag{C.12}$$

This is still positive for  $\alpha = 1/2$ .

### D. FI terms in the toroidal case

The main aim of this appendix is to fix the sign of the FI term with respect to the charge of the  $\varphi$  fields. To do this, we will compute the mass of the  $\varphi$ 's from the purely stringy formulae and check they consistently come from a FI term. Then we check that the corresponding FI term computed by other means agrees with this calculation. The content of this section is based on ref. [19].

#### D.1 From the mass formula

Consider a factorisable  $T^6 = (T^2)^3$  whose basis of 1-forms we denote by  $dx^i, dy^i, i = 1, 2, 3$ . A IIA factorisable D6 brane in such toroidal setup is described by six wrapping numbers<sup>20</sup>

$$D6 \quad (n^1, m^1) \quad (n^2, m^2) \quad (n^3, m^3),$$

where  $n_i (m_i)$  represents the number of times the brane wraps the  $x^i (y^i)$  direction. Given two stacks of D6 branes that intersect at several points in the  $T^6$ , there will be a single massless chiral fermion living in each one of the intersection points. The net number of intersection points gives thus the net number of chiral fermions, that is given by the absolute value of

$$I_{ab} = \prod_{i=1}^3 (n_a^i m_b^i - n_b^i m_a^i). \tag{D.1}$$

For each one of these intersection points there will be a tower of scalars, of which the lightest ones have masses

$$m_{1,ab}^2 = \frac{1}{2\alpha'} (-|\theta_{ab}^1| + |\theta_{ab}^2| + |\theta_{ab}^3|) \quad m_{2,ab}^2 = \frac{1}{2\alpha'} (|\theta_{ab}^1| - |\theta_{ab}^2| + |\theta_{ab}^3|) \tag{D.2}$$

$$m_{3,ab}^2 = \frac{1}{2\alpha'} (|\theta_{ab}^1| + |\theta_{ab}^2| - |\theta_{ab}^3|) \quad m_{4,ab}^2 = \frac{1}{\alpha'} \left[ 1 - \frac{1}{2} (|\theta_{ab}^1| + |\theta_{ab}^2| + |\theta_{ab}^3|) \right], \tag{D.3}$$

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<sup>20</sup>See [33] for a review on intersecting branes.

where  $\theta_{ab}^i$  is the angle<sup>21</sup> the brane  $a$  makes with the brane  $b$ , in units of  $\pi$ .

For arbitrary wrapping numbers, a D6 will go to a (stack of) D9(s) with magnetic flux under 3 T-dualities along the  $x$  directions, but special choices of wrapping numbers can lead us to lower dimensional (stacks of) branes. We can use this D6-brane language to describe D7 branes, as follows. A  $D7_i$  is defined as a D7 that is pointlike in the  $i^{\text{th}}$  torus and wraps completely the  $j^{\text{th}}$  and  $k^{\text{th}}$  ones. After the three T-dualities we can describe a single  $D7_i$  by the numbers

$$\begin{array}{llll} D7_1 & (1, 0) & (n_2^1, 1) & (n_3^1, -1) \\ D7_2 & (n_1^2, -1) & (1, 0) & (n_3^2, 1) \\ D7_3 & (n_1^3, 1) & (n_2^3, -1) & (1, 0) \end{array}$$

Consider the intersection between a  $D7_i$  and a  $D7_j$  where we only turn on magnetic flux in the torus where both D7 branes overlap (that is,  $n_i^j = n_j^i = 0$  for these particular  $i, j$ ). Then it is easy to see that there are two lightest scalars, whose mass is

$$m_{ij}^2 = \frac{1}{2\pi\alpha'} \left| \arctan \left( \frac{(2\pi)^2 \alpha'}{A_k} n_k^i \right) + \arctan \left( \frac{(2\pi)^2 \alpha'}{A_k} n_k^j \right) \right| \simeq \frac{2\pi |n_k^i + n_k^j|}{A_k}. \quad (\text{D.4})$$

This mass can never be tachyonic. On the other hand it is clear that since whenever  $n_k^i + n_k^j \neq 0$  we will have a chiral fermion living between both branes, the presence of this relative magnetic flux is breaking supersymmetry. We can see that the mass of this scalar can be seen as coming from a couple of FI terms, each one of them associated to one of the stacks of branes and, in particular, to the presence of magnetic flux on them. The FI term associated to a  $D7_i$  brane when magnetic flux is present in the  $k^{\text{th}}$  torus is easily computable and given by

$$\xi_i = \frac{2\pi n_k^i}{A_k} = \frac{\int_{T_k^2} F}{A_k}. \quad (\text{D.5})$$

Since the squared mass (D.4) is always positive, one can see that the sign of the FI term is correlated with the charge of the bifundamental scalars under the gauge groups. A straightforward analysis shows that whenever one has two  $D7$ s that overlap over a  $T^2$  where both of them have flux, the charge under  $U(1)_i$  of the state going from the brane  $i$  to the brane  $j$  has the same sign as the FI term associated with the brane  $i$ .

Now we provide more evidence that this is indeed a FI term, following methods already developed in papers [19, 22], so we will be brief.

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<sup>21</sup>Given the wrapping numbers  $(n_i^j, m_i^j)$  with all the  $n_i^j \geq 0$ , this angle is given by  $\pi\theta_{ab}^i = \tan^{-1} \left( \frac{m_i^b R_i^y}{n_i^b R_i^x} \right) - \tan^{-1} \left( \frac{m_i^a R_i^y}{n_i^a R_i^x} \right)$ . This is the situation usually stressed in the literature. However, one has to take into account that when negative values for one or both of the  $n_i^j$  are used the formula for this angle is slightly different, though straightforward to obtain in a case-by-case analysis.

**D.2 From the DBI action**

Another way of checking that magnetic flux gives rise to a FI term is to consider the difference between the DBI action for the D7-brane with magnetic flux and this same action in the absence of flux. The DBI action in the Einstein frame reads (notice the absence of curvature contributions in the toroidal case)

$$S = -T_p \int d^{p+1}x \sqrt{g_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}}, \tag{D.6}$$

with

$$T_p = \frac{(2\pi)^{-p}\alpha'^{-(p+1)/2}}{g_s}. \tag{D.7}$$

From this expression we read the gauge coupling constant (in the supersymmetric limit) for the U(1) living in the D7:

$$\frac{1}{g_i^2} = \frac{V_4}{(2\pi)^5 g_s \alpha'^2} \tag{D.8}$$

with  $V_4$  the (dimensionful) volume wrapped by the D7. Now, note that the difference in vacuum energy in the four dimensional theory due to the presence of the magnetic flux is

$$\delta V = \frac{(2\pi)^{-7}V_4}{g_s \alpha'^4} \left( \sqrt{1 + (2\pi\alpha')^2 \left( \frac{2\pi n_k^i}{A_k} \right)^2} - 1 \right) \simeq \frac{1}{2} \frac{1}{g^2} \left( \frac{\int_{T_k^2} F}{A_k} \right)^2 \tag{D.9}$$

in the dilute flux approximation. Note that equations (D.5) and (D.9) can consistently come from a D-term potential of the form

$$V_{D,i} = \frac{g_i^2}{2} \left( \frac{\xi_i}{g_i^2} + \sum_j q_j |\phi_j|^2 \right)^2 \tag{D.10}$$

where  $\phi_j$  stands for all the canonically normalised fields charged under the corresponding U(1) with  $q_j$  being their corresponding charges, and  $\xi_i$  is given by (D.5).

**D.3 From the  $N = 1$  supergravity algebra**

FI terms can be extracted from the Kähler potential  $K$  of the compactification, via the supergravity formula

$$\frac{\xi_i}{g_i^2} = \left( \frac{\partial K}{\partial V^i} \right)_{V^i=0} \tag{D.11}$$

where  $V^i$  is the vector superfield of the corresponding U(1). In toroidal compactifications the Kähler potential for closed string moduli reads (both in Type IIA and Type IIB)<sup>22</sup>.

$$K/M_P^2 = -\log(S + S^*) - \sum_{i=1}^3 \log(T_i + T_i^*) - \sum_{i=1}^3 \log(U_i + U_i^*), \tag{D.13}$$

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<sup>22</sup>Note a different normalisation of the Kähler potential with respect to e.g. [19]. In particular

$$(M_P^2)_{\text{[ours]}} = \frac{1}{8\pi} (M_P^2)_{\text{[19]}}. \tag{D.12}$$

where (in Type IIB)

$$\begin{aligned} \text{Re } S &= \frac{1}{2\pi g_s} \\ \text{Re } T_i &= \frac{A_j A_k}{(2\pi)^5 g_s \alpha'^2} \end{aligned} \tag{D.14}$$

The exact formula for the  $U$  fields will be irrelevant in what follows. Now, suppose a  $D7_i$  brane wraps the four-cycle whose volume is parametrised by  $T_i$ . Suppose we turn on worldvolume magnetic field in the  $j^{\text{th}}$  2-torus,  $j \neq i$ . Then, following the arguments developed in [22] for the general case, the  $T$  fields that will become charged will be the ones corresponding to the 4-cycles that intersect the 4-cycle wrapped by the  $D7_i$  in a 2-cycle threaded by magnetic flux. In this case, it is easy to see that, since the torus has no self-intersections, if a  $D7_i$  has magnetic flux in the  $j^{\text{th}}$  2-torus, then only the 4-cycle whose volume is parametrised by  $T_k$  will become charged ( $i \neq j \neq k$ ). That implies that one must modify the Kähler potential (D.13) making the change  $T_k \rightarrow T_k - Q_k V_i$ , where  $Q_k$  is the charge associated to this cycle, so that it remains a gauge invariant function of the superfields [18]. Applying formula (D.11) we obtain

$$\frac{\xi_i}{g_i^2} = M_P^2 \frac{Q_k}{2 \text{Re } T_k}. \tag{D.15}$$

Using the fact that  $1/g_i^2 = \text{Re } T_i$  and that

$$\frac{M_P^2}{2} = \frac{(2\pi)^{-7} V_6}{g_s^2 \alpha'^4} \tag{D.16}$$

with  $V_6$  being the dimensionful compactification volume, we obtain, equalling (D.5) and (D.15)

$$Q_k = \frac{(2\pi)^7 g_s^2 \alpha'^4}{V_6} \text{Re } T_k \text{Re } T_i \frac{\int_{T_j^2} F}{A_j} = \frac{1}{(2\pi)^3} \int_{T_j^2} F, \tag{D.17}$$

so the D-term potential reads

$$V_{D,i} = \frac{1}{2 \text{Re } T_i} \left( \frac{1}{4\pi^2} \frac{n_j^i}{2 \text{Re } T_k} + \sum_a q_a |\phi_a|^2 \right)^2 \tag{D.18}$$

in accordance with [22].

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